OPEN ACCESS
International Journal of Advanced Economics
P-ISSN: 2707-2134, E-ISSN: 2707-2142
Volume 4, Issue 5, P.No. 86-106, June 2022
DOI: 10.51594/ijae.v4i5.335
Fair East Publishers
Journal Homepage: www.fepbl.com/index.php/ijae

NEURAL NETWORK ANALYSIS AND GAUSSIAN MIXTURE MODELS IN PREDICTING ENVIRONMENTAL PSYCHOLOGICAL IMPACTS ON ECONOMIC GROWTH

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Article Received: 15-05-22 **Accepted:** 01-06-22 **Published:** 08-06-22

ABSTRACT

The study aimed to examine the environmental psychological impact on economic growth. Specifically, the study explored how individuals' choices/selections influence the trade-offs of economic growth and environmental quality. The cross-sectional survey data were randomly sampled from 211 individuals in two regions in Tanzania. The data analysed by using neural network analysis and Gaussian Mixture models. This study found that the environment has a negative impact on economic growth. The study concluded that the nature of the relationship between economic growth and environmental quality is preliminary (fundamentally) determined by both levels of income (economic growth) and environmental psychological well-being of the individual imposed by the available environmental instruments such as policies and regulations. The study recommended that to achieve both environmental quality and economic growth; the government and other stakeholders should design and implement the environmental instruments that maximise the environmental psychological well-being of the individual.

Keywords: Environmental Psychological Impacts, Economic Growth, Neural Network Analysis and Gaussian Mixture Models.

INTRODUCTION

The natural environment or sometimes called natural capital plays a fundamental role in our economy as a direct input into production and through many services it provides. Environmental resources (renewable and non-renewable) such as forests, water, mineral, and fossil fuels are directly facilitating the production of goods and services. The role of the natural environment can be either provisioning services, regulating services, cultural services, or supporting services (Everett, Ishwaran, Ansaloni, and Rubin, 2010). Only the role of provisioning services, the products obtained from the ecosystem have market prices, e.g., ores and minerals. Many other ecosystem services provide benefits outside of markets, therefore, measures of economic activities such as a GDP do not capture the full benefits provided to us by the natural environment nor do they reflect the extent to which environmental resources have been depleted or degraded (Everett et al. 2010). Notably, the natural environment contributes to economic output through two main channels. First, a direct input to the process of economic activity (e.g., renewable resources such as forests and fisheries, and non-renewable resources such as minerals, fossil fuels, etc). Second, indirect effects on the productivity of other factors of production through global life support functions such as climate, water, chemical composition regulations of the atmosphere, pollution filtering, and waste sink (Everett et al. 2010). Therefore, to safeguard or maintain sustainably the environmental economic benefits, the government and other stakeholders should set (well-design) and implement effective economic-based environmental instruments such as policies, regulations, and social awareness programmes. Thus, the relationship between economic growth and the environment will be explained by those economic-based environmental instruments.

Empirical literature depicts that the common nature of the relationship between economic growth and environmental quality is an inverted U-shaped curve (Kuznets, 1955; Panayotou, 1997; 2003; Aung, Saboori, and Rasoulinezhad, 2017; Ekins, 1997), inverted J shaped curve (Selden and Song, 1995; Panayotou, 1997), J shaped curve (Andree, Chamorro, Spencer,…, and Dogo, 2019), N-shaped and inverted N-shaped curve (Ozokcu and Ozdemir, n.d), and U shaped curve (Bundala, Ngaruko, and Lyanga, 2021; Jia and Shen, 2017; Yan 2012; Jingyan and Lishe, 2010). Specifically, this study examined the driving forces or nature of the U-shaped curve/function and inverted U-shaped curve /function.

One of the most attractive concepts is the environmental Kuznets curve (EKC) which explains the inverted U-shaped hypothesis. There are mixed facts on the policy relevance of the EKC hypothesis. According to Acaravci and Akalin (2017), the EKC hypothesis is not valid in developing countries. Moreover, Everett et al.(2010) evidenced the relevance of the EKC hypothesis to policy-makers is limited as its analysis was based on a limited set of pollutants; it does not apply to all types of environmental damages. In addition, the environmental Kuznets relationship appears strongest for pollutants with a significant local impact, for example, carbon and other greenhouse gases on the other hand, where the impact is globally and diffuse, emissions have continued to rise with the increase in income per capita even in the richest countries (Everett et al. 2010). Phimphanthavong (2013) emphasised that the EKC hypothesis is fundamentally a within-country story. On the other hand, most studies support the policy relevance of the EKC hypothesis in their local domains /countries; therefore the EKC hypothesis lacks its generalized policy implication (Tiba and Omari, 2016; Everett et al.2010; Kozluk and Zipperer, 2015; Lin and Swanson, 2010; Kumar, 2020). Recent studies in their localities (countries), evidenced the U-shaped curve of the relationship between environment and economic growth (Bundala, Ngaruko, and Lyanga, 2021; Jia and Shen, 2017). In addition, Dogan and Inglesi-Lotz (n.d) evidenced the U-shaped curve in the European countries.

Truly, the study of the relationship between economic growth and the environment is complex as it operates through several different channels such as preferences, technology, and economic structure (Shafik, 1994). In other words, the relationship depends on the size of the economy (sectoral structure), the vintage of the technology, and the demand for environmental quality (Galeotti, 2003). Everett et al. (2010) identified three drives of the economy-environmental relationship that are scale effects based on the expansion of economic activities, composition effects based on economic structure, and technical effects based on the technological substitution effects on the production process. Moreover, they emphasised the change in preference of society may also drive changes in the environmental damage, for example, through encouraging changes in the stringency of environmental regulation of the industry. Therefore, the rationale of environmental policy is to manage the provision and use of environmental resources in a way that supports continued improvements in the prosperity and well-being of current and future generations. Thus, the reason for government policy intervention is due to market failures as the nature of the natural environment is both full or partial public goods and the existence of externalities.

Deductively, the relationship between economic growth and environmental quality is preliminarily determined by the individual/society's preferences on (need for) environmental quality as public goods. Therefore, a demand for the environmental quality of the individuals can be reflected in the individuals' willingness to comply with the current policy and regulations which are a consequence of environmental awareness (Shafik, 1994; ECLAC, 2000; Max and Jiang, 2019). Sensibly, environmental awareness as a new policy tool, in addition to legal and economic instruments changes people's behaviour. Until recently people's awareness was never considered a possible tool to promote environmental policy. However, this tool is important and has the potential to be a powerful tool in the environmental sphere (ECLAC, 2000). In addition, environmental issues have a wide range of impacts on both quantity and quality of labour through diseases from water and air pollution. Therefore, the early implementation of tight environmental regulations could harm economic growth and increase environmental damage in the long run (Everett et al. 2010). In other words, environmental regulation may reduce the productivity of the firms in the regulated sectors. The benefit of environmental regulation is to induce change in the firm behaviour, particularly in the longer term. In addition, a well-designed regulatory instrument generally enables companies to seek innovative solutions that otherwise would remain unexplored (Porter and van der Linde, 1995; Porter, 1991).

Some studies suggest that the useful guideline for designing environmental policy to minimise any unavoidable trade-offs between environmental and economic policy goals are market-based proportionality (e.g., including safety margin), cost-effectiveness consideration, and applicability or operational simplicity (Kozluk and Zipperer, 2015; Porter and van der Linde, 1995; Porter, 1991). Moreover, indirect effects of the well-designed environmental regulatory instruments might induce firms to innovate, which in turn might increase productivity and hence profitability–potentially outweighing the increases in abatement cost (Porter hypothesis). Shafik(1994) concluded that policies that reflect social decisions about the provision of environmental public goods depend on the sum of individual benefits relative to the sum of the individuals' willingness to pay, therefore, the environmental problems can be externalised to minimise the externalities effects (Lennox, Harris and Codur, 2019; Shafik, 1994; Jorgenson, Goettle and Wilcxen, 2013; Drews, Jeroen and Bergh, 2017). People's awareness and support for solving environmental problems are likely to depend on whether they are directly affected by the issues (Kim and Lee, 2018). On the other hand, environmental problems may directly impact economic growth through a restriction or reduction in production, and adversely affects production factors, or indirectly through higher emission reduction costs (Tiba and Omari, 2016; Emerton, Karanja, and Gichere, 2001; Jia and Shen, 2017). It is suggested that to increase awareness and support for addressing environmental problems caused by rapid economic growth in developing countries, helping individuals internalise the costs of environmental damage originating in a remote area through information and training seems essential (Kim and Lee, 2018; Sepehrdoust and Zamani, 2017; Alege and Ogundipe, 2013). Vogel (2018) concluded that the fear of adverse environmental problems contributes to the positive environmental policy compliance of the individuals.

In general, the literature evidence that the relationship between economic growth and environmental quality is determined by the psychological alertness (willingness) of the individual on complying with environmental instruments such as policies, regulations, and social awareness programmes. Therefore, the complexity of the relationship is evidenced by the various relationships represented by letter-shaped curves such as inverted and non-inverted U, N, and J-shaped curves. Relying on this fact, the study aimed to examine the environmental psychological impact on economic growth. Specifically, the study explored how individuals' choices/selections influence the trade-offs of economic growth and environmental quality. Furthermore, the study examined the factors that determined either U-shaped functional or/and inverted U-shaped functional relationship between economic growth and environmental quality. As the literature cleared that the non-linear relationship is exhibited. Therefore, the study analysed the data by using neural network analysis and Gaussian Mixture models to fill the methodological gap resulting in the literature on the contradictory findings of EKC hypothesis evidenced.

METHODOLOGY

A study used the cross-section survey. The data was sampled from two administrative regions in Tanzania, the Mwanza and Kagera regions. The sample size was 211 individuals, randomly sampled from four districts in Mwanza and Kagera regions. The Tabachnick and Fidell (2019) approach was used to estimate the sample size. The self-checklist questionnaires were used to collect the data by self-administered methods. The consent and confidentiality of research ethics were adhered to. The independent variables are indicators of psychological environmental factors which are environmental/social awareness (Soa), environmental policy (Pol), environmental sustainability (Su), and environmental regulation (Re). The 5-points Likert was used to establish the construct score and its observed variables. The dependent variable is economic growth (average GDP per capita) which is a proxy of the annualised monthly income of the individual.

The study analysed the data by using two non-linear analytic tools. The neural network analysis and Gaussian mixture models were used to examine the empirical implications of psychological environmental awareness in economic growth. The two common neural network architectures are Multilayer Perceptron (MLP) and radial basis function network (RBF). Moreover, the Gaussian function was applied in modelling the basic function in the RBF (Bishop, 1994; Santos, Rupp, Bonzi, and Fileti, 2013).

To understand how the input data which are explanatory variables (Sus, Soa, Pol, and Re) are mapped to the output which is a dependent variable (economic growth) through non-linear activation function g (.), the neural network analysis was established.

Now, let input data X (our independent variables) be distributed as $x_1, x_2, x_3, ..., x_d$, and they are weighted by a parameter W as $w_1, w_2, w_3, ..., w_d$ respectively. Therefore, the total weighted input signals (φ) to the processing unit (neuron) is the summation of the input data and their respective weights (Bishop, 1994). That is,

$$\varphi = \sum_{i=1}^{d} w_i x_i + w_0 x_0 \tag{1}$$

Where $w_0 x_0$ is the constant value indicating the signal impact of the extra value of input data/variable when it is set at$x_0 = 1$, the equation, becomes

$$\varphi = \sum_{i=0}^{d} w_i x_i \tag{2}$$

Then, this total signal impact of the input data is non-linearly activation function g(.) so that the output z_j for a single layer (hidden layer or neuron) will be determined as,

$$z_j = \mathrm{g}(\varphi) \tag{3}$$

$$z_j = \mathrm{g}\left(\sum_{i=0}^{d} w_{ji} x_i\right) \tag{4}$$

Therefore, if the output in the first layer z_j is further transformed to the next (second) layer through weights $\breve{w}_{kj}$ for hidden unit *j* and output unit *k* and new input z_j and activated by non-linearly function $\breve{g}$ (.) to the second output y_k, when z_0 =1

$$y_k = \breve{g}\left(\sum_{j=0}^{m} \breve{w}_{kj} z_j\right) \tag{5}$$

When, z_j is substituted in the equation, y_k becomes

$$y_k = \breve{g}\left(\sum_{j=0}^{m} \breve{w}_{kj}\, \mathrm{g}\left(\sum_{i=0}^{d} w_{ji} x_i\right)\right) \tag{6}$$

To enable our input data X to vary from -1to 1, i.e., $-1 \leq X \leq 1$, since training the network requires a differentiable mapping function, thus a sigmoidal (S-shaped) activation function is applied. Therefore, the input layer activation function is a hyperbolic function (tanh);

$$\mathrm{g}(\varphi) = \tanh\varphi = \frac{e^{\varphi} - e^{-\varphi}}{e^{\varphi} + e^{-\varphi}} \tag{7}$$

And its first derivative $\mathrm{g}'(\varphi)$ is expressed as

$$\mathrm{g}'(\varphi) = 1 + \mathrm{g}(\varphi)^2 \tag{8}$$

The output (Y) is expected to vary from 0 to 1 as the probability of economic growth, therefore the appropriate output layer activation function is the logistic sigmoid given by;

$$\mathrm{g}(\varphi) = \frac{1}{1 + \mathrm{e}^{-\varphi}} \tag{9}$$

And its first derivative $\mathrm{g}'(\varphi)$ is expressed as

$$\mathrm{g}'(\varphi) = g(\varphi)\{1 - \mathrm{g}(\varphi)\} \tag{10}$$

Noticing that for the linear output, the linear activation function (identity) is used, this is expressed as,

$$\mathrm{g}(\varphi) = \varphi \tag{11}$$

Furthermore, a study applied the radial basis function (RBF) neural network architecture to examine how the dependent variables (economic growth) $y(x)$ are mapped by sub-population function or sub-models that are defined by the input data (X) in the population (Bishop, 1994). The Gaussian mixture models/function was used to define the basis function, $\Phi_\mathrm{j}(x)$. Therefore, the output of the network x input of k-unit $y_k\,(x)$ is determined as a linear superposition of basis functions, in the training set, that is,

$$y_k(x) = \sum_{j=0}^{m} \breve{w}_{kj}\Phi_j\,(x) \tag{12}$$

Whereby, $\Phi_j(x)$ is a radial symmetric function centred on the j[th] data point, which signifies the activation of hidden layer j when the network is represented with input vector *x*. Where *m* is the number of basis functions. The bias for the output layer has been represented as an extra basis function Φ_0 which its activation function is fixed at $\Phi_0 = 1$. The common basis function is the Gaussian distribution function, $\Phi_\mathrm{j}(\mathrm{x})$ which is expressed as,

$$\Phi_\mathrm{j}(\mathrm{x}) = ae^{\frac{-(x-\mu_j)^2}{2\sigma^2}} \tag{13}$$

Where a is the height of the curve's peak given by $\frac{1}{\sqrt{2\pi\sigma^2}}$, and μ the position of the centre of the peak (mean). Therefore, the basis function is determined by the components of the mixture model for the distribution of input data, and then the Gaussian Mixture models (GMM) were applied in the training set. That is, the probability density, *P(x)* is expressed as a linear combination of the basic function in the form of a GMM (Yu and Sapiro, 2011).

$$P(x) = \frac{1}{m}\sum_{j=1}^{m} \frac{1}{(2\pi)^{\frac{d}{2}}\sigma_j^d}\Phi_j(x) \tag{14}$$

Where the pre-factor in front of $\Phi_j(x)$ is chosen to ensure that the probability density function integrates to unity, that is $\int P(x)dx = 1$.

The study mapped the input data into probability outcomes, i.e., categorised the economic growth (dependent variable) to go high (1 score) or low (0 scores). Therefore, we need to take a vector of real-valued arguments and transform it into a vector whose elements fall in the range (0, 1) and sum to 1. That is, the Softmax activation function, $S(x)_i$ in the hidden layer was used, and the identity action function on the output layer (Bishop, 1994; Santos et al. 2013).

$$S(x)_i = \frac{\exp(x_i)}{\sum_{j=1}^{n} \exp(x_j)} \tag{15}$$

A suitable error function was defined concerning a set of data points, and the parameters (weights) are chosen to minimize the error, therefore, the sum-of-squares error (SSE) function was used. It is defined as the squares of individual errors summed overall output units and all patterns. Therefore, the error function (E) for neural networking training is presented as,

$$E = \frac{1}{2}\sum_{q=1}^{n}\sum_{k=1}^{c}\left\{y_k(x^q; w) - t_k^q\right\}^2 \tag{16}$$

Where x^q is an input vector and t^q a target vector for output k, when the network is presented with a pattern q.

The data pre-processing involved the rescaling of scale-dependent variables which the normalized values used, which fall between 0 and 1. It is given by (x−min)/ (max−min) (Haykin, 1998; Ripley, 1996; Gurney, 1997).

This is the required rescaling method for scale-dependent variables if the output layer uses the sigmoid activation function. In particular, the values 0 and 1, which occur in the uncorrected formula when x takes its minimum and maximum value, define the limits of the range of the sigmoid function but are not within that range. The corrected formula is [x− (min−ε)]/[(max +ε)−(min−ε)]. Specify a number greater than or equal to 0 (Haykin, 1998; Gurney, 1997).

The neural network was trained by a Mini-batch. Divides the training data records into groups of approximately equal size, then updates the synaptic weights after passing one group; that is, mini-batch training uses information from a group of records. Then the process recycles the data group if necessary. Mini-batch training offers a compromise between batch and online training, and it may be best for "medium-size" datasets (Ripley, 1996). Moreover, the gradient descent optimization algorithm is used to minimize the error function (Gurney, 1997; Santos et al. 2013). For the RBF architecture, the activation function for the hidden layer is the radial basis function, which "links" the units in a layer to the values of units in the succeeding layer. For the output layer, the activation function is the identity function; thus, the output units are simply weighted sums of the hidden units. Therefore, the ordinary radial basis function uses the exponential activation function so the activation of the hidden unit is a Gaussian “bump” as a function of the inputs (Bishop, 1994). The dimensionality reduction analysis was done by principal component analysis (PCA) (Bishop, 1994). The Kaiser-Meyer-Olkin measure of sampling adequacy test (KMO) tests whether the partial correlations among variables are small. Bartlett’s test of sphericity tests whether the correlation matrix is an identity matrix, which would indicate that the factor model is inappropriate. The study used a varimax method, which is an orthogonal rotation method that minimizes the number of variables that have high loadings on each factor (Ugulu, 2013). This method simplifies the interpretation of the factors.

RESULTS

Data Cleansing

The pre-principal component analysis test was done by Kaiser-Meyer-Olkin Measure of Sampling Adequacy (KMO) and Bartlett's Test of Sphericity. The test of KMO was 0.881 which is within the recommended value of at least 0.50 (Ugulu, 2013). This indicates that the data is adequate for factors analysis. Moreover, Bartlett's Test of Sphericity has a chi-square of 1105.509, a significance level of 0.000 which is less than a critical value of 0.01 indicating the presence of inter-correlation of the items or variables in the construct (Table 1).

Table 1

KMO and Bartlett's Test of Sphericity for Envi construct

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		0.881
Bartlett's Test of Sphericity	Approx. Chi-Square	1105.509
	df	6
	Sig.	0.000

Source: Author (2021)

Table 1 shows the KMO and Bartlett's Test of Sphericity as pre-factor analysis tests. The table evidenced that the data sample was adequate for conducting factor analysis, and there is an inter-correlation of the construct items. Hence the principal component analysis (PCA) was done to examine the construct validity (discriminant and convergent validity). The component matrix was established by using PCA under the Varimax rotation (Table 2).

Table 2

Component Matrix[a] for Envi construct

	Component
Items/observed variable	1
Pol	0.968
Soa	0.959
Re	0.953
Sus	0.947

Extraction Method: Principal Component Analysis.
a.1 Components extracted.

Source: Author (2021).

Table 2 shows the component matrix of the PCA of the Envi construct. The table evidenced that there is only one component with the factor loadings that range from 0.947 to 0.968 which are within the recommended value of at least 0.50 (Hair, Risher, Sarstedt, and Ringle, 2019). All the items /variables converged to one construct and discriminated from another construct. Therefore, both divergent and discriminant validity was ensured. That is, there is one construct (Envi) and its observed/manipulated psychological variables (MPVs) are Sus, Soa, Pol, and Re. Furthermore, the Gaussian modelling was done to examine the distributional behaviour of the data. The Gaussian Mixture Models (GMM) were used to examine the data classes in the study and evidenced that two main data classes were explained by the MAP classification (Figure 1).

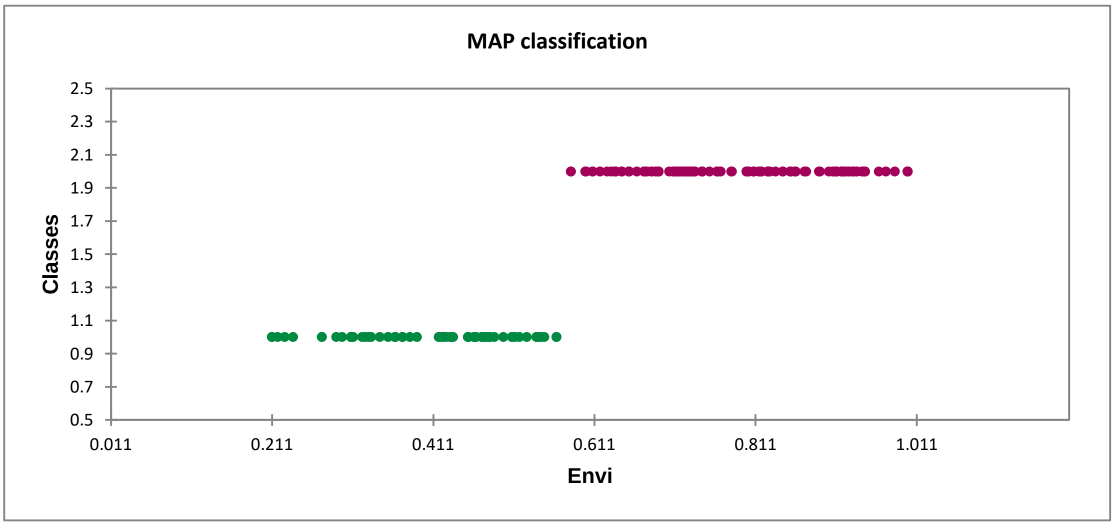

Figure 1: The MAP classification of the Envi construct in the population sample
Source: Author (2021)

Figure 1 shows the MAP classification of the Envi data distribution sampled from Mwanza and Kagera regions. Class one is classified by an Envi from 0.211 to 0.611 and is composed of 66 respondents equal to 31.28 percent, and class two is composed of 145 respondents equal to 68.72 percent. From GMM the sampled population was most weighted from the Envi score of 0.611 to 1.0 for class two. Further modelling was done by establishing the fitted model for mixture models for classes 1 and 2 (Figure 2).

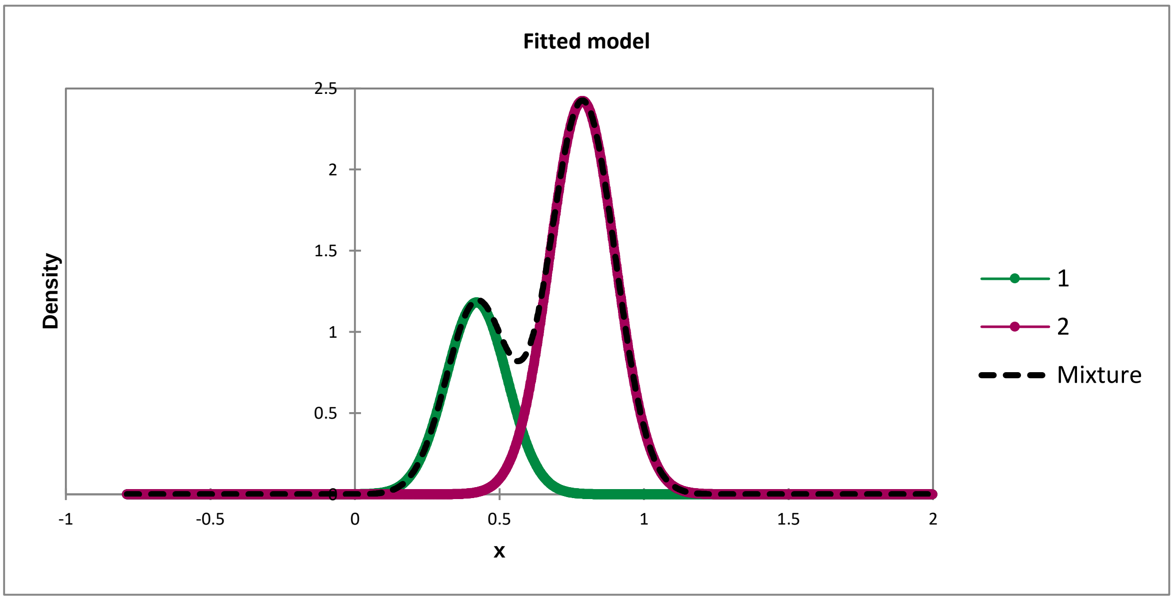

Figure 2: The Fitted model for classes one and two of the Envi distributional models
Source: Author (2021).

The post-analysis of the classes describes that the class has been characterised by an average GDP per capita of 1.376 TZS million, with a minimum of 0.2832 TZS million to 4.368 TZS million. Class 1 has an average range of MPVs scores from 0.3938 to 0.4531 and class 2 has a range of 1.75 TZS millions with a range of 0.354 to 4.992 TZS millions of the GDP per capita (Table 3& 4).

Table 3

Descriptive Statistics for Class 1

Variable	Obs	Mean	Std. Dev.	Min	Max
AGDP	68	1.376276	.6864803	.2832	4.368
Sus	68	.4530882	.1498283	.2	.93
Soa	68	.4107353	.1336661	.2	.6
Pol	68	.3938235	.1137952	.2	.73
Re	68	.4313235	.1491259	.2	.8

Source: Author (2021).

Table 4

Descriptive Statistics for class 2

Variable	Obs	Mean	Std. Dev.	Min	Max
AGDP	143	1.748291	.8133368	.354	4.992
Sus	143	.7899301	.1437648	.2	1
Soa	143	.7765734	.1490302	.4	1
Pol	143	.7987413	.1203274	.6	1
Re	143	.7995105	.1187987	.47	1

Source: Author (2021).

Table 3 & 4 shows the descriptive statistics for class 1 and 2. The averaged MPVs scores for class 2 range from 0.7899 to 0.7995. Class 1 and 2 have 68 and 143 respondents respectively. The NEC criterion is greater than 1; hence, there is no clustering structure in the data of Envi (Yu and Sapiro, 2011).

Multi-Layer Perceptron of the Envi

The multilayer perceptron (MLP) was done by supervised training with an algorithm known as “error back propagation” to examine the non-prescribed or non-assumed relationship between economic growth and MPVs of the psychological environmental factors (Envi). A binary classification problem may have a single output neuron and use a sigmoid activation function to output a value between 0 and 1 to represent the probability of predicting a value for class 1 (Gurney, 1997; Santos et al. 2013). This can be turned into a crisp class value by using a threshold of 0.5 and snap values less than the threshold to 0 otherwise to 1 (Santos et al.2013; Ripley, 1996).

The model summary for MLP analysis disclosed that the Sum of Squares Error (SSE) of the training data was 2.003 and that of testing data was 0.481, and the relative error is 0.716 for the testing sample (Table 5)

Table 5

Model Summary of Multilayer Perceptron

Training	Sum of Squares Error	2.003
	Relative Error	0.888
	Stopping Rule Used	1 consecutive step(s) with no decrease in error[a]
	Training Time	0:00:00.04
Testing	Sum of Squares Error	0.481
	Relative Error	0.716
Dependent Variable: AGDP		
a. Error computations are based on the testing sample.		

Source: Author (2021)

Table 5 shows the model summary of the Multilayer Perceptron of the Envi construct. The relative error for the training sample was 0.888, and the stopping rule was 1 consecutive step(s) with no decrease in error which the computation is based on the testing sample. The input layer consists of the covariates (independent variables) of Sus, Soa, Pol, and Re, and they are rescaled with a standardized method. The hidden layers are activated by the hyperbolic tangent function (Table 6).

Table 6

Network Information for Multiplayer Perceptron (MLP) for the Envi construct

Input Layer	Covariates	1	Sus
		2	Soa
		3	Pol
		4	Re
	Number of Units[a]		4
	Rescaling Method for Covariates		Standardized
Hidden Layer(s)	Number of Hidden Layers		2
	Number of Units in Hidden Layer 1[a]		3
	Number of Units in Hidden Layer 2[a]		2
	Activation Function		Hyperbolic tangent
Output Layer	Dependent Variables	1	AGDP
	Number of Units		1
	Rescaling Method for Scale Dependents		Normalized
	Activation Function		Sigmoid
	Error Function		Sum of Squares

a. Excluding the bias unit

Source: Author (2021)

Table 6 shows the MLP network information for the Envi construct. The MLP output layer (dependent variable) is AGDP and rescaled by the normalised method. The activation function is the Sigmoid and the error function is the sum of squares. The MLP architecture was shown to describe the neuron inter-connection and its synaptic weights (Figure 3).

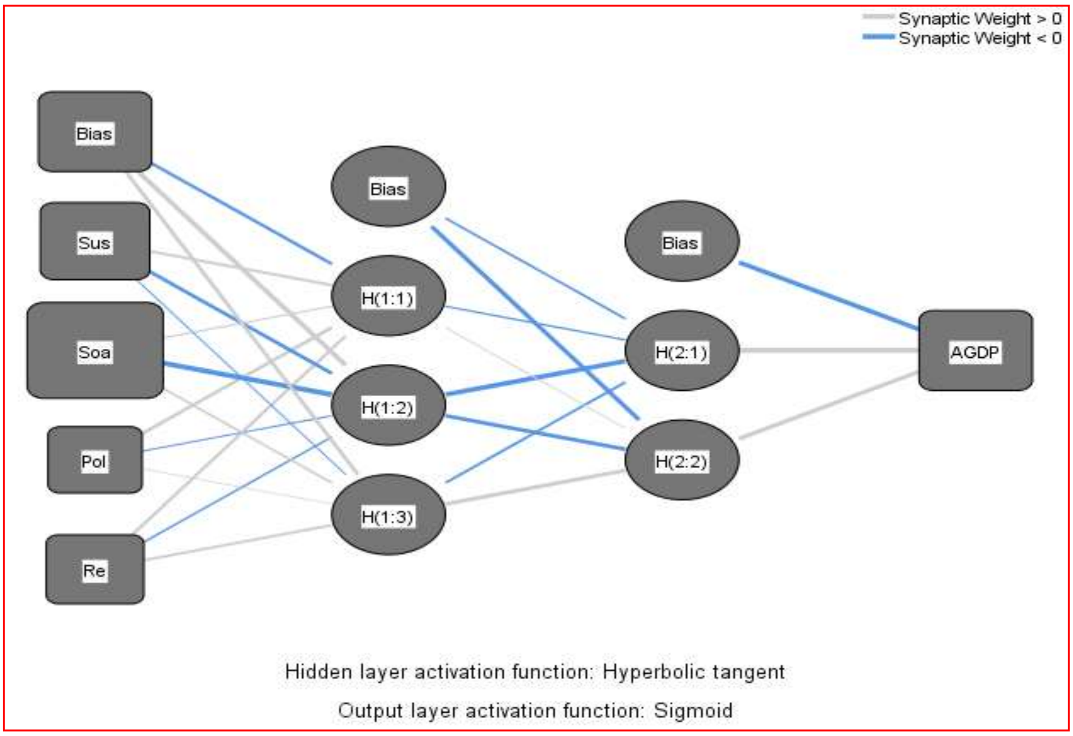

Figure 3: The Forward Feed MLP Architecture for Envi Construct

Source: Author (2021).

Figure 3 shows the MLP architecture (network of neurons) of the Envi construct. The architecture describes the forward relationship between economic growth (output) and psychological environmental factors (input). The synaptic weights for node or neurons H (2:1), and H (2:2) are less than zero, indicating that the input is not able to predict economic growth. The Soa has a negative effect and an important impact on economic growth at H (2:2). The economic growth was positively predicted by another unknown predictor (input) determined by the bias of the output which is positive and important.

Radial Basis Function

The radial basis function (RBF) was done for examining the sub-models or classes of the output layer (high or low economic growth). The Gaussian mixture model was used to activate the RBF. The advantage of RBF over MLP is the use of the pre-classified or categorical output that is determined by the softmax function (Yu and Sapiro, 2011; Ripley, 1996; Santos et, al. 2013). The model summary was described (Table 7).

Table 7

The Radal Basis Function (RBF) Model Information

Training	Sum of Squares Error	2.003
	Relative Error	0.868
	Training Time	0:00:00.20
Testing	Sum of Squares Error	0.497[a]
	Relative Error	0.748

Dependent Variable: AGDP

a. The number of hidden units is determined by the testing data criterion: The "best" number of hidden units is the one that yields the smallest error in the testing data.

Source: Author (2021).

Table 7 shows a model summary of the RBF. The SSE of the training sample was 2.003 and the relative error was 0.868. The SSE of the testing was 0.497 and the relative error was 0.748. The input layer (covariates) are Sus, Soa, Pol, and Re (independent variables). The rescaling method of covariates was standardised. The hidden layer has four units and was activated by Softmax. The output layer has a one-unit (dependent variable) the economic growth (AGDP). The output was normalized and activated by the identity function. The error function was the sum of squares (Table 8).

Table 8

Radial Basis Function (RBF) Network Information

Input Layer	Covariates	1	Sus
		2	Soa
		3	Pol
		4	Re
	Number of Units		4
	Rescaling Method for Covariates		Standardized
Hidden Layer	Number of Units		4[a]
	Activation Function		Softmax
Output Layer	Dependent Variables	1	AGDP
	Number of Units		1
	Rescaling Method for Scale Dependents		Normalized
	Activation Function		Identity
	Error Function		Sum of Squares

a. Determined by the testing data criterion: The "best" number of hidden units is the one that yields the smallest error in the testing data.

Source: Author (2021).

Table 8 shows the RBF network information for the Envi construct. The RBF architecture was established to show the neuron network built by the basis function of Gaussian mixture models (Figure 4).

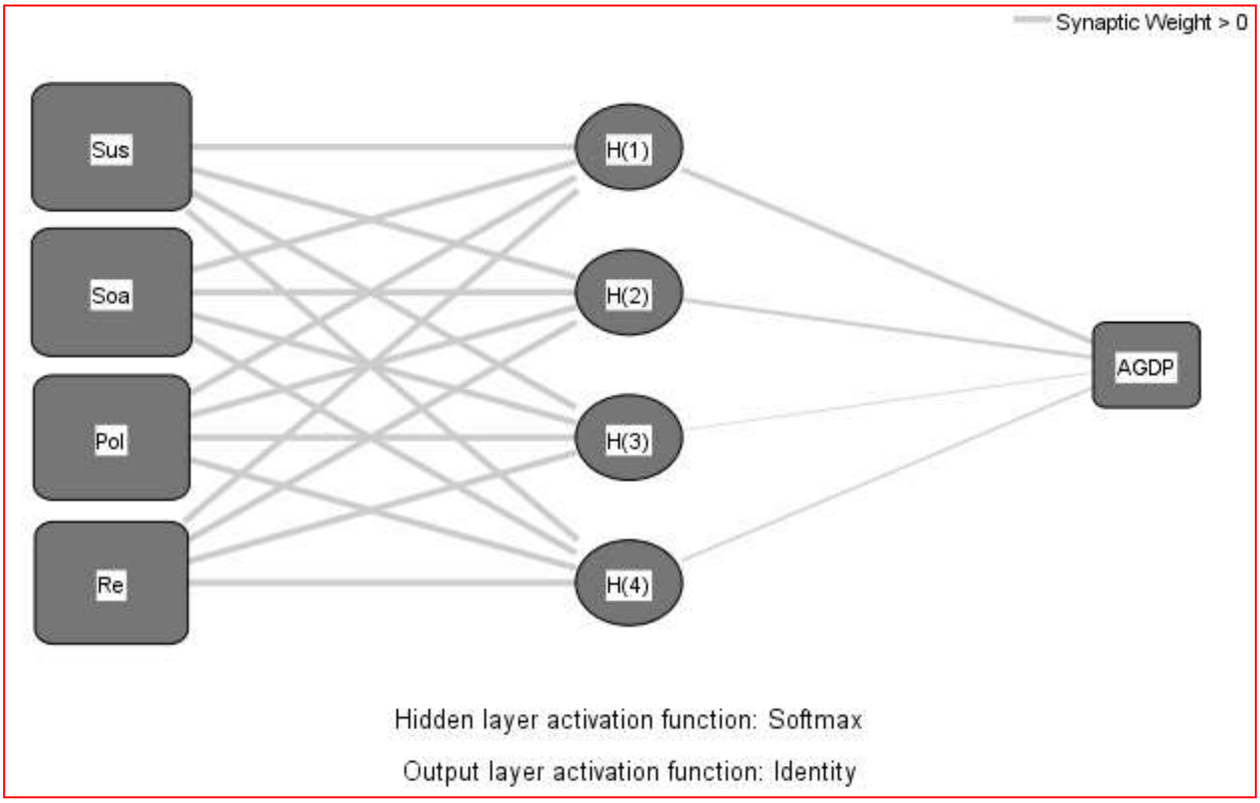

Figure 4: The RBF architecture for Envi construct
Source: Author (2021).

Figure 4 shows the RBF architecture of the Envi constructs that explain the predictive output of economic growth. The covariates (inputs) were activated through Softmax function and the output layer was activated by the identity function. The model depicts that all the covariates or the Envi construct have negatively impacted economic growth. They have negative synaptic weights but are important predictors of economic growth. This finding coincided with the finding of MLP. That is, the environment has a negative impact on economic growth.

Ex Facto Post-Analysis of the Finding

Further examination was done on the relationship between economic growth and psychological environmental factors (Envi). The basic or primary finding indicates that the Envi has a negative impact on economic growth. Therefore, further examination of the relations behaviour was required. That is, the curve estimation was done. Each item of the construct (independent variables) was related in two-way curves. The curve examination discloses the full relationship of each item/variable to economic growth. The curve examination of the environmental sustainability (Sus) and economic growth was established in each class. Class one which is characterised by the low average economic growth of 1.376TZS million and psychological environmental sustainability (PES) index of 0.4731 has evidenced the EKC hypothesis of the inverted U-shaped curve (Figure 5).

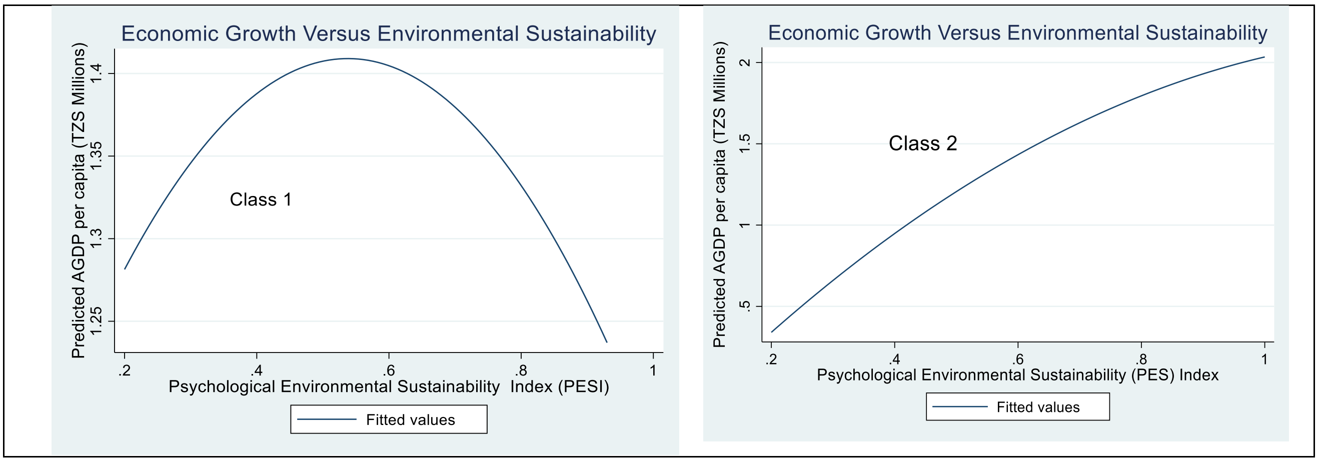

Figure 5: The Curve Estimation of the Economic Growth and Environmental Sustainability

Source: Author (2021).

Figure 5 shows the two-way relationship between economic growth and psychological environmental sustainability. Class one is characterised by the relatively less economic growth of average at 1.376 TZS million and PES index of 0.453 than class two. The inverted U-shaped curve is exhibited. This implies that the countries with lower economic growth (developing countries) exhibit the EKC hypothesis which is contrary to Acaravci and Akalin (2017) who suggests that, EKC is not relevant to developing countries. Class two has a relatively higher economic growth of 1.748 TZS million and PES of 0.790 than class one. In class two the concavity relationship between economic growth and the psychological environmental sustainability index is evidenced. The concavity function evidences the diminishing marginal return principle. In other words, in class two the insignificant linear relationship between economic growth and psychological environmental sustainability is exhibited. This means that the population that has both high economic growth and psychosocial environmental awareness their economies are hardly shifting altogether with their psychological awareness. Notably, class two has lesser economic growth than class one.

Moreover, the curve relation of psychological environmental social awareness (Soa) and economic growth was established. Class one which is characterised by a relatively low mean GDP per capita of 1.376 TZS millions and the psychological environmental social awareness (PESA) index or psychosocial environmental awareness (PEA) index of 0.4107, exhibits a concavity relationship between economic growth and PESA index. On the other hand, class two which is characterised by a relatively higher economic growth of 1.748 TZS millions on average and a PESA index of 0.7766, exhibits a convexity function relationship between economic growth and environmental social awareness (Figure 6).

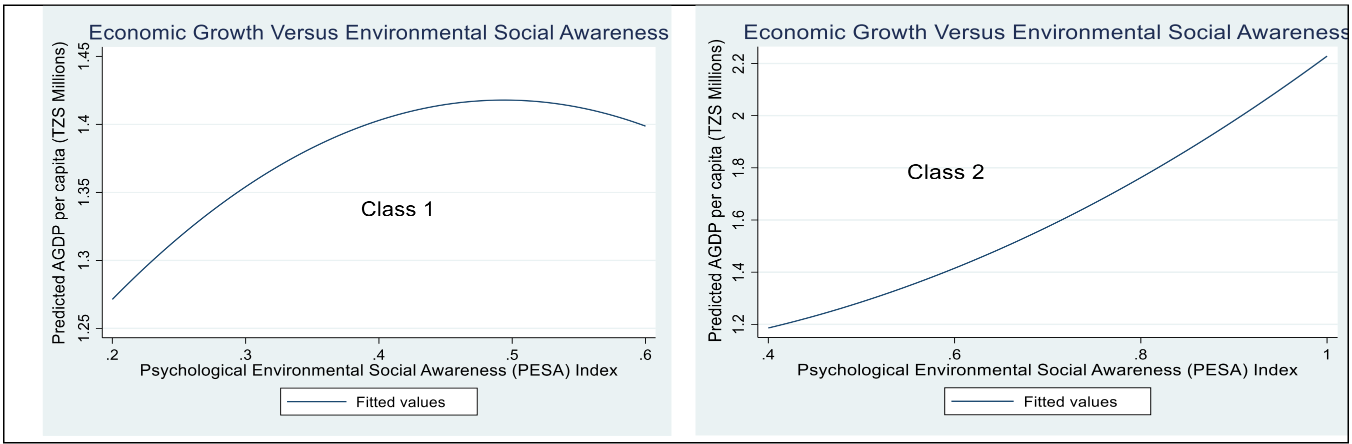

Figure 6: The Curve Estimation of the Economic Growth and Environmental Social Awareness

Source: Author (2021).

Figure 6 shows the curve estimation of economic growth and environmental awareness. The concavity relationship between economic growth and environmental social awareness evidenced the economic rules of scarcity and choice that are determined by the principle of opportunity costs in economic planning. The concavity evidences the diminishing marginal return principle as further PESA index is consumed. That is, class one, with a relatively lower economic growth and psychological environmental awareness, PESA is described as an input in the production of the output of economic growth (individual income). Class two which is characterised by relatively higher economic growth and higher psychological environmental awareness has a non-significant linear impact on economic growth. it exhibited a convexity functional relationship between economic growth and PESA.

Furthermore, the curve estimation was done to explore the nature of psychological environmental policy and economic growth. Class one which is a relatively lower economic class has a psychological environmental policy (PEP) index of 0.3938. This class has an inverted-J-shaped curve or partial–inverted U-shaped curve. This is due to the distribution data effect (DDE). The curve indicates a short period of the positive impact of the low psychological environmental awareness; in the long run, the policy awareness is associated with the low economic growth (Figure 7).

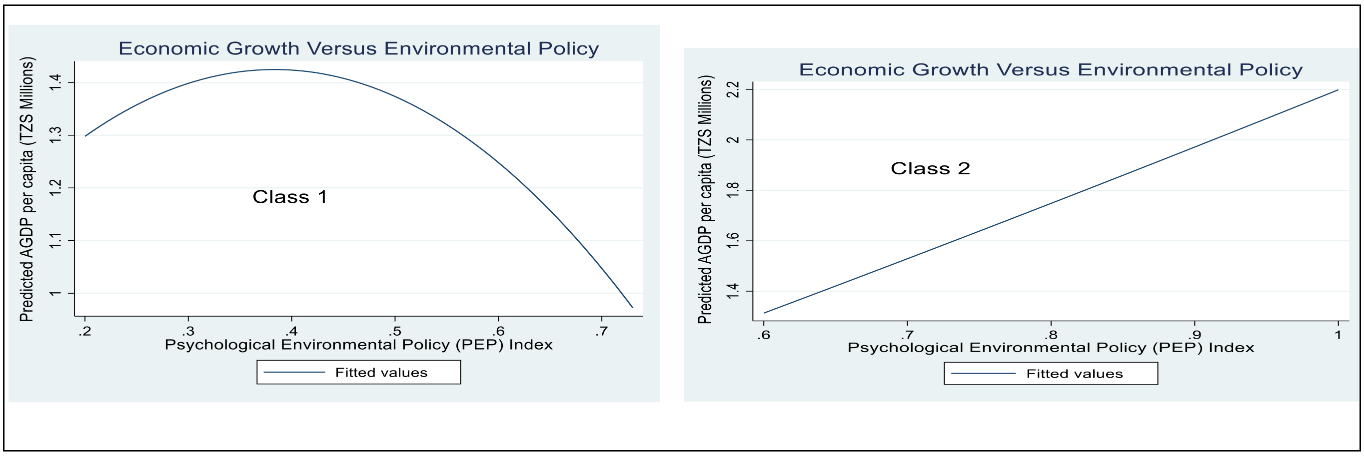

Figure 7: The Curve Estimation of the Economic Growth and Psychological Environmental Policy

Source: Author (2021).

Figure 7 shows the curve estimation of the economic growth and psychological environmental policy. In class two which is a class with a relatively higher economic growth than class one, and a PEP index of 0.7987 on average, the linear relationship between economic growth and PEP index was exhibited. Furthermore, the curve estimation of the economic growth and environmental regulations was done. The EKC hypothesis was evidenced in class one with low economic growth and psychological environmental regulations index of 0.4313 (Figure 8)

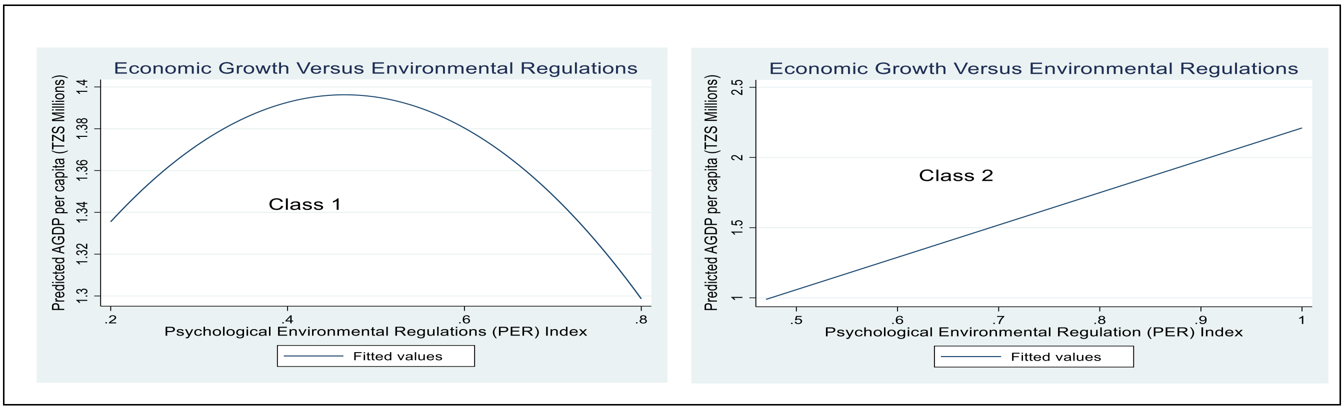

Figure 8: The Curve Estimation of Economic Growth and Psychological Environmental Regulations

Source: Author (2021).

Figure 8 shows the curve estimation of the economic growth and psychological environmental regulations. Class two which is characterised by a relatively higher economic growth and PER index of 0.7995 has evidenced the linear relationship between economic growth and the psychological environmental policy.

The lesson learned from the curve estimation is the fact that the nature of the latter-shaped curves like inverted U-shapes, J-shaped, and others are due to the data distribution effect (DDE) on the modelling. The study evidenced two facts from the two different populations that have different measured characteristics. More specifically, the study evidenced that in a population characterised by low economic growth and low psychosocial environmental awareness the relationship between economic growth and the environment tends to be either inverted U-shaped or J-shaped curves.

Discussion of Findings

The impact of the environment on economic growth or economic growth on environmental quality is debated by many researchers under the popular environmental hypothesis of EKC introduced by Kuznets (1955). The recent studies tried to find the empirical validity of the environmental Kuznets curve (EKC) hypothesis. The finding contradicted in a great way. Some studies support the EKC hypothesis (Kuznets, 1955; Panayotou, 1997; 2003; Aung, Saboori, and Rasoulinezhad, 2017; Ekins, 1997). On the other side, some researchers disagree with the generalisation of the EKC hypothesis (Selden and Song, 1995; Panayotou, 1997; Andree et al. 2019). The position of this study evidenced that the relationship between environmental quality and economic growth is relayed on the data distributional models/ effects. In other words, the relationship depends on the level of psychological well-being on environmental issues and economic growth. Therefore, we can generalise the nature of the environmental relationship with economic growth relative to the level of willingness of an individual to adhere to the environmental instruments-policies and regulations and other social programmes that maximise the social awareness of the individual. It can be generalised that the impact can positive (significant or insignificant) or negative (significant or insignificant) which can be described by the either inverted or not inverted U and J-shaped curves.

This study confirmed that the environment has a negative impact on economic growth as its synaptic weights of the MLP and RBF were negative. The most negative significant predictor is the psychological environmental social awareness which has a negative impact on the economy but is important. The curve analysis indicates that environmental social awareness (Soa) has a concave function with economic growth. That is the increases of the Soa result in diminishing returns of production input (Soa). In class one, the average of Soa is about 0.62 which is above the input value of 0.50 (Soa) of the maximum output. That is why the negative impact relationship existed which was constrained by the level of economic growth.

The environmental regulation and environmental sustainability in class one, the class that is characterised with a relatively lower economic growth than class two, exhibited the EKC hypothesis; it evidenced the inverted U-shaped curve relations. That is, a society with a low income is less demanding of the quality of the environment. In other words, society is likely or willing to incur more costs for demanding the quality of the environment or maintain the environmental-related costs when its relative income is higher. Clearly, at a later stage when the income is high the individual preference shifts to environmental quality and is likely to spend more on demanding the quality of the environment. At this stage, society is aware of the environmental quality and has income to spend for demanding the quality of the environment. That is, a society with a low income will not likely incur costs to demand environmental regulation and maintain sustainability, because of the preference for income. But when society gets high income, the trade-off between income and demand for quality of environment will be minimised, and therefore, an individual or society started to follow or demand the regulations and need for sustainability of the environment, because society's preference is for quality of the environment. Based on this study, a negative relationship was found, which means the society of Tanzania is still at the early stage of trading-off between the quality of the environment and income demand. The environmental policy and the economic growth are related negatively, but it has an inverted J-shaped curve or partial-inverted U-shaped curve. The J-shaped curve indicates the reality that the environmental policy should not be stringent the economic growth. That is, a low or moderate psychological environmental policy index is required for the better economic performance of the society or individual. Thus, this study confirms the study by Selden and Song (1995), Panayotou (1997), and Andree et al. (2019) on the J-shaped curve. In other words, the J-shaped curve is exhibited because the society or an individual choice or prefer the environment as public goods (necessity good) that have a limited amount or fixed value of satisfaction. This study goes in line with Smulder (1995) who concluded that the environmental policy affects growth, both in the long run and in the short run, by affecting the productivity of investment and the savings behaviour of consumers. Therefore, this study supports that the environment provides necessary inputs to economic production and accumulation processes. Hence improvements in environmental quality that follow the environmental policy may boost the productivity of the environment and growth (Chen, Shieh, and Chang, 2015; Smulder, 1995).

One of the impressive findings of this study is that the nature of the relationship between economic growth and environmental is varying from negative (significant or insignificant) to positive (significant or insignificant), which is described by both inverted and non-inverted U and J –shaped curves. The EKC hypothesis is not generalisable.

CONCLUSION AND RECOMMENDATIONS

Based on the fact that the relationship between economic growth and environmental quality depends on the preference or willingness of the individual to “consume” or demand the “quality” of the environment, therefore, the fundamental determinants of the relationship between them are the environmental psychological well-being of an individual and level of income (economic growth). That is, the positive environmental psychological well-being and income level of an individual are key determinant factors for the relationship between economic growths. For example, the environmental policy or regulations that maximise the environmental well-being of the individual would be supported by many, as the result, the environmental quality will be achieved in parallel to the economic growth. The stringent policies and regulations on environmental issues will hurt economic growth because they reduce the morale or willingness of the individual to adhere to the environmental policies and regulations.

Empirically, the study evidenced that the relationship between environmental quality and economic growth was not limited or defined only by the EKC hypothesis; it is going beyond that fact. It is determined highly by two fundamental factors, the income level (level of economic growth) and the environmental psychological well-being of individuals imposed by the environmental regulation, policies, and other social programmes that maximise the willingness of the individual to demand the environmental quality. Therefore, the study concluded that the nature of the relationship between the economic growth and environmental quality is preliminary (fundamentally) determined by both levels of income (economic growth) and environmental psychological well-being of the individual imposed by the available environmental instruments such as policies and regulations, and other social programmes that increases the environmental social awareness and sustainability demanding.

The study poses one recommendation, to achieve both environmental quality and economic growth; the government and other stakeholders should design and implement the environmental instruments that maximise the environmental psychological well-being of the individual. For example, the design of the environmental policies and regulations that maximise the willingness of the individual on demanding the needs or costs of the quality of the environment would be encouraged. For example in Tanzania, the National Environmental Policy (NEP) 1997 realised the importance of environmental social awareness as it set as one of its main policy objectives (URT, 1997). However, there is no strong mechanism and policy commitment to controlling and managing demographic dynamics concerning changing their mindset (Maro, 2008; Pallangyo, 2007). Therefore, this study has a policy implication for NEP 1997.

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